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$$P.(R\mp\rho\sin\epsilon)=Q(r\pm\rho\sin\epsilon)\pm W\rho\sin\epsilon,$$

where R, r, and ρ are the radii of the wheel, the axle, and their common axis respectively, and ϵ is the limiting angle of resistance.

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DIOPHANTINE ANALYSIS.

Conducted by J. M. COLÁW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

16. **Proposed by H. W. DRAUGHON, Olio, Mississippi.**

Find three numbers such that the cube of any one plus the sum of the squares of the other two, will be a square.

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Let ax, bxy, and cxz represent the required numbers; then we have $a^3x^3+(b^2y^2+c^2z^2)x^2=\square \dots (A)$, $b^3x^3y^3+(a^2+c^2z^2)x^2=\square \dots (B)$, and $c^3x^3z^3+(a^2+b^2y^2)x^2=\square \dots (C)$. Omitting in (A), (B), and (C), the factor x^2, and putting $x=2bcyz/a^3$, we have (A) as a perfect square.

Substitute this value of x in the first term of (B); then, obviously, the condition that (B) will be a perfect square, is $2b^4cy^4z/a^3=2acz$.

$\therefore y=a/b \dots (1)$. After performing a similar operation in (C), we obtain $2bc^4yz^4/a^3=2aby$. $\therefore$ $z=a/c \dots (2)$.

Consequently $x=2bcyz/a^3=2/a \dots (3)$; and the required numbers are $ax=2$, $bxy=2$, and $cxz=2$.

[NOTE.—Can any of our contributors find three *unequal* numbers answering the conditions of this problem? The proposer and several contributors have reported that they had as yet failed to solve it. The problem seems difficult of solution, or at least the EDITOR does not now see any way clear to a solution of it.]

17. **Proposed by ARTEMAS MARTIN, LL.D., U. S. Coast and Geodetic Survey Office, Washington, D. C.**

Is it possible to find two positive whole numbers such that each of them and also their sum and difference, when diminished by unity shall all be squares?

Solution by the PROPOSER.

Let x^2+1 and y^2+1 denote the numbers required; then their sum $=x^2+y^2+2$, their difference $=x^2-y^2$, and we have

$$x^2+y^2+1=\square=v^2\ldots\ldots(1),$$
$$x^2-y^2-1=\square=w^2\ldots\ldots(2).$$

From the first of these equations, $y^2=v^2-x^2-1\ldots\ldots(3)$. Adding (1) and (2) we get $2x^2=v^2+w^2\ldots\ldots(4)$. Let $v=t+u$, $w=t-u$ and (4) becomes $x^2=t^2+u^2\ldots\ldots(5)$, which is satisfied by

$$t=p^2-q^2,\ u=2pq,\ x=p^2+q^2, \text{ and then } v=p^2+2pq-q^2.$$

Substituting these values of x and v in (3) we get

$$y^2=4pq(p^2-q^2)-1\ldots\ldots(6).$$

As the right-hand member of (6) is of the form $4m-1$ it can not be an integral square, and therefore the problem is impossible.

II. **Solution by C. A. ROBERTS, Long Bottom, Ohio.**

Let (x^2+1) and (y^2+1) be the numbers, which when diminished by 1 give x^2 and y^2.

Let $(x^2+1)-(y^2+1)-1=a^2=x^2-y^2-1$, which call (I);

Let $(x^2+1)+(y^2+1)-1=b^2=x^2+y^2+1$, which call (II).

The square root of an even square, is even; the square root of an odd square, is odd. Take (I), $a^2=x^2-y^2-1$, and transposing, $a^2+y^2+1=x^2$ or three squares whose sum is a square. The square of any even number is divisible by 4, without a remainder, and is therefore said to be of the form of $(4n)$. If the square of any odd number be divided by 4, there will be a remainder of 1, and such squares are said to be of the form of $(4n+1)$. Any number not of the form of $(4n)$ or $(4n+1)$ is not a square. Let us determine in the equation $a^2+y^2+1=x^2$ whether a and y, are both odd, both even, or one of them odd and the other even.

1 is odd and of the form of $(4n+1)$; if a is even and y odd, or if y is even and a odd, we have for the form of the sum $(4n)+(4n+1)+(4n+1)$ $=(12n+2)$, which is of the form of $(4n_1+2)$ and which can not be a square. If a and y are both odd, we have for the form of the sum, $(4n+1)+(4n+1)$ $+(4n+1)=12n+3$, which is of the form of $(4n_1+3)$, and which can not be a square. If a and y are both even, we have for the form of the sum, $(4n)+(4n)+(4n+1)=(12n+1)$, which is of the form of $(4n_1+1)$, and may be a square, and if $(4n_1+1)$ is a square, as x^2, it is an odd square, and x is odd. Therefore in order that the equation $a^2+y^2+1=x^2$ shall be true in integers, a and y *must* be even numbers, and x *must* be an odd number.

Take (II) $x^2+y^2+1=b^2$; applying to (II) the reasoning in (I) x and y *must* be even, and b odd, or x must be both odd, (as in I) and even, (as in II). As this is impossible with the same value of x, there are no such numbers as called for in the problem.

[Notes.—Chas. de Medici of 60 West 22nd St., New York, overlooking the punctuation of the problem as published, gives 64 and 81 as the numbers, and adds an interesting exhibit showing that the curio of these values are not by any means limited to what the question, as he read it, asked for.

M. A. Gruber, P. H. Philbrick, and G. B. M. Zerr, should have been credited for solving problem 15, December Number. Their solutions were selected for publication, but owing to the fact the December Number had to be cut short in order to get it out without further delay their solutions were omitted.—EDITOR.]

PROBLEMS.

25. **Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.**

Find, if possible, integral values of each of the seven linear measurements of a rectangular parallelopiped; i. e. length, breadth, height, the diagonals of each of the three different rectangular sides, and the diagonal from an upper corner to the opposite lower corner; or, find integral values, if possible, of a, b, c, d, e, f, and g, as shown in the equations,—$a^2+b^2=c^2$, $a^2+d^2=e^2$, $a^2+f^2=g^2$, $b^2+d^2=f^2$, $b^2+e^2=g^2$, $c^2+d^2=g^2$, $c^2+e^2=f^2$. If not possible, how many of them can have integral values? and which?

26. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Find (1) a *square fraction* the arithmetical difference of whose terms is a *cube*; and (2) find a *cubic fraction* the arithmetical sum of whose terms is a *square*.

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AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

12. **Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

A large plane area is ruled by two sets of parallel equidistant straight lines, the one set perpendicular to the other. The distance between any two lines of the first set is a; the distance between any two lines of the second set is b. If a regular polygon of $2n$ sides be thrown at random upon this area, find the chance that it will fall across a line, the diameter of the circum-circle of the polygon being less than a or b.

II. **Solution by H. W. DRAUGHON, Clinton, Louisiana.**

In the rectangle $ABCD$ let $AB=a$ and $AC=b$. Let c=apothem of polygon, and r=radius of its circum-circle.

Let the sides of the rectangle $EFIJ$, be parallel to, and distant c, from the corresponding sides of $ABCD$, and let the sides of the similarly